A Markov chain approach on pattern of rainfall distribution

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A Markov Chain Approach On Pattern of Rainfall Distribution

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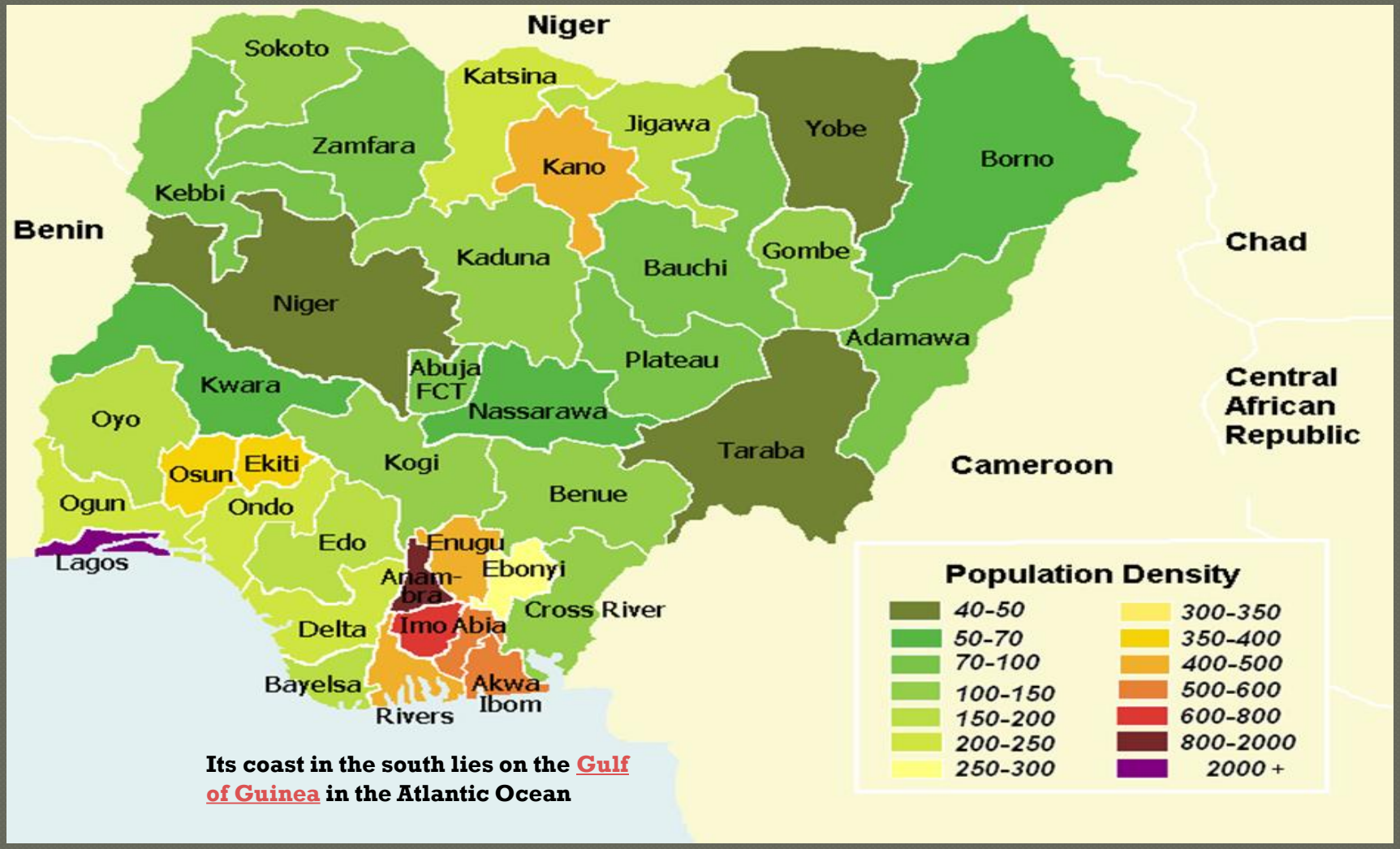
NIGERIA



Its coast in the south lies on the Gulf of Guinea in the Atlantic Ocean

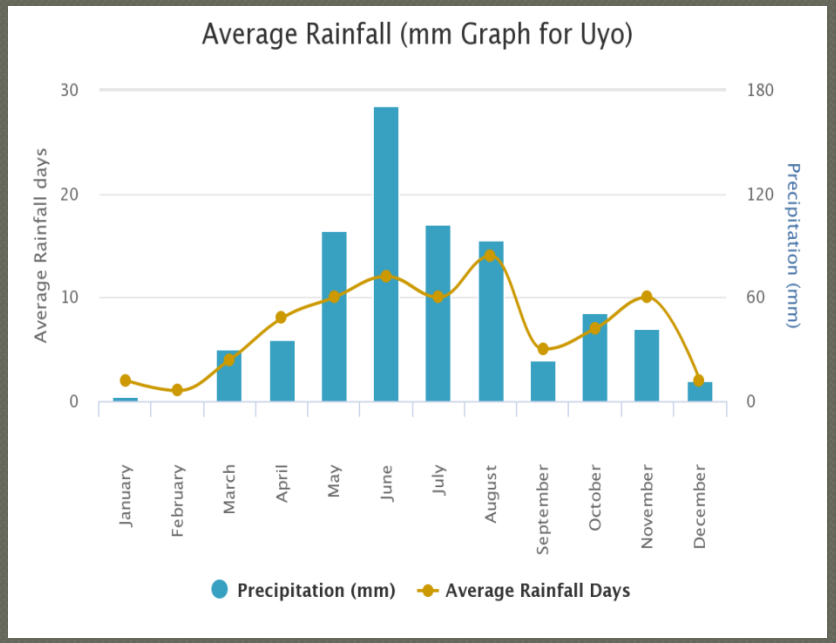
UYO Versus EKET

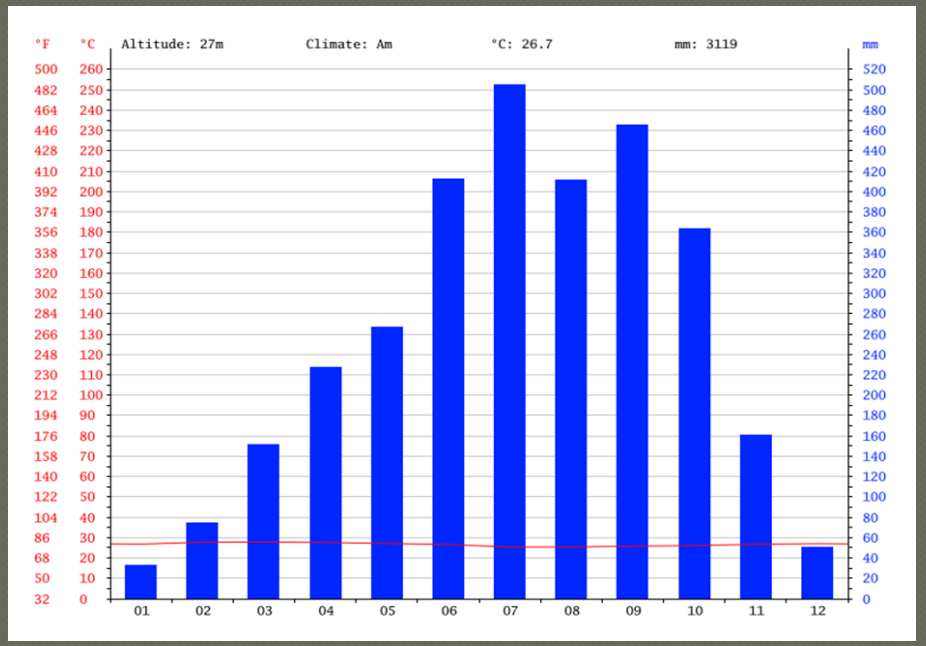
- Uyo is the state capital of Akwa Ibom Population of about 882, 779 people

- **Eket** is the second largest city in Akwa Ibom, with a total population of 564, 489

(2006 Census)

Plots of RAINFALL statistics: UYO & EKET





EKET:

The driest month is January, with July having the highest precipitation records

It is highly difficult, if not impossible, to forecast what the weather will be like at a certain time in a very precise place

Brief Introduction

- Rainfall exhibits Strong variability in time and space across the globe.
- Its stochastic modelling is necessary for the prevention of natural disaster
- Demand for rainfall to life has made its study a major focus to many researchers Mark (2005) Umoh, Akpan and Jacob (2013)

Motivation: Benefits of Rainfall

- Examining rainfall's Distribution and Pattern of extreme high and low Precipitation is good:
  - For the Agriculture and the Economy of many African countries including Nigeria
  - For determining best adapted Plant Species and the Optimum time for Planting
  - For future planning
  - For general Growth of a Nation

Motivation: Negative Impacts

- The Hydrological extremes driven by rainfall include:
  - Floods- farm and communities

Motivation: Negative Impacts

- Droughts and Landslides

2012 Nigeria's floods Scenario

- Debris and Mud-flows

- https://www.youtube.com/watch?v=OU_YY7cN7n8

Model: Established Literature

- Many Probability Models developed in studies such as:
- Manning (1950);
- Feyerherm and Bark (1967);
- Kulandaivelu 1984;
- Phien and Ajirajah 1984;
- Topalogu 2002

Literature (Markov Models)….

- Aneja and Srivastava (1986, 1999) apply two-state (with two parameters) and three-state (with five independent parameters) models

- Purohit, Reddy, Bhaskar, and Chittora (2008) apply two-state Markov chain model to obtain probabilities of occurrence of dry and wet weeks

- **Garg and Singh (2010)** examine patterns of rainfall at Pantnagar for daily rainfall data of 42 years (1961-2002) using a three-state Markov chain model.

Methods &Design

Data: (15 YEARS)

- UYO-Daily 1st Jan, 1995 to 31st Dec, 2009

Seasons:

(1)Pre-Monsoon **(Jan 1-March 31)**; (2)Monsoon **(April 1 - September 30) ;**
(3)Post-Monsoon (**Oct,1-Dec, 31)**

- Regimes (States)-dry (d), wet (w) and rainy (r)
- Classifications: $\leq 2.50mm - dry$; $2.51mm \leq R < 5.0mm - wet$; $\geq 5.0mm - rainy$

Data & Methods

Data: (15 years)

- EKET-Daily 1st Jan1993- 31st Dec.,2008
- Seasons: Monthly Grouped
- Regimes (States)-dry (d) and rainy (r)
- Classifications: $\leq 2.50mm - dry; \geq 2.51mm - rainy$

Methodology

- A Markov chain

a branch of stochastic process where the state space and index (time) are discrete

A discrete-time process where future process behaviuor depends purely on the immediate past (Udom, 2010)

		Current Day (j)			Total
		Dry (d)	Wet (w)	Rainy (r)	
Previous Day (i)	Dry (d)	n_{dd}	n_{dw}	n_{dr}	$n_{d.}$
	Wet (w)	n_{wd}	n_{ww}	n_{wr}	$n_{w.}$
	Rainy (r)	n_{rd}	n_{rw}	n_{rr}	$n_{r.}$

Methodology……

- The Maximum Likelihood Estimators of $P_{ij}, i, j = \{d, w, r\}$

$\hat{p}_{ij}$ are given by $\hat{p}_{ij} = \dfrac{n_{ij}}{\sum_{j=d}^{r} n_{ij}}$

		Current Day		
		Dry(d)	Wet(w)	Rainy(r)
Previous Day	Dry (d)	P_{dd}	P_{dw}	P_{dr}
	Wet (w)	P_{wd}	P_{ww}	P_{wr}
	Rainy (r)	P_{rd}	P_{rw}	P_{rr}

- ***Goodness-of-Fit Test***

  Wang and Maritz (1990) $WS = \dfrac{S_a + S_b - 1}{\sqrt{V(S_a + S_b - 1)}} \xrightarrow{P} N(0, 1)$

Methodology….

- ***Long Run Probabilities***:

$$\begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\end{bmatrix} = (p_1\ p_2\ p_3)\begin{pmatrix}P_{dd} & P_{dw} & P_{dr}\\ P_{wd} & P_{ww} & P_{wr}\\ P_{rd} & P_{rw} & P_{rr}\end{pmatrix}$$

Expected Length of Different Spells of Seasons and Weather Cycle (WC)

- The expected length of dry spell is given by

$$E(D) = \frac{1}{1-P_{dd}}$$

- ***Weather Cycle (WC): The weather cycle of each of the periods is given by***

$$\boldsymbol{E(WC) = E(D) + E(W) + E(R)}$$

RESULTS: UYO

- **Estimated Values of WS Test Statistic and the associated p-values**

	Pre-Monsoon	Monsoon	Post-Monsoon
WS Statistic value	2.26 (p=0.0119)	11.04 (p < 0.0001)	15.77 (p < 0.0001)

- **Estimated Equilibrium State Probabilities, Expected Length of different Season's Spell, Weather Cycle and Total Number of days**

Period	Equilibrium state probability			Expected length of Season's Spell			Weather Cycle	Total No. of Days
	Dry (π_1)	Wet(π_2)	Rainy (π_3)	Dry Spell	Wet Spell	Rainy Spell		
Pre-Monsoon	0.89	0.02	0.09	10.0	1.0	1.0	12.0	1363
Monsoon	0.56	0.08	0.36	2.0	1.0	2.0	5.0	2744
Post-Monsoon	0.77	0.05	0.18	6.0	1.0	1.0	8.0	1380

RESULTS: EKET

- Probability of Rainfall Distribution per Month per season

Months	Initial Matrix (p)	Long Term Prob.	Prob. for either rain/dry on a day	
	P	p^n	P(dry)	P(rains)
January	$\begin{bmatrix}0.71 & 0.29\\ 0.61 & 0.39\end{bmatrix}$	$P^9\begin{bmatrix}1.000 & 0\\ 0.999 & 0.0001\end{bmatrix}$	1.000	0
February	$\begin{bmatrix}0.91 & 0.09\\ 0.82 & 0.18\end{bmatrix}$	$P^8\begin{bmatrix}0.9011 & 0.0989\\ 0.9011 & 0.0989\end{bmatrix}$	0.9011	0.0989
March	$\begin{bmatrix}0.741 & 0.26\\ 0.63 & 0.37\end{bmatrix}$	$P^5\begin{bmatrix}0.7079 & 0.2921\\ 0.7079 & 0.2921\end{bmatrix}$	0.7079	0.2921
April	$\begin{bmatrix}0.60 & 0.40\\ 0.63 & 0.37\end{bmatrix}$	$P^6\begin{bmatrix}0.6117 & 0.3883\\ 0.6117 & 0.3883\end{bmatrix}$	0.6117	0.3883
May	$\begin{bmatrix}0.45 & 0.55\\ 0.52 & 0.48\end{bmatrix}$	$P^6\begin{bmatrix}0.4860 & 0.5140\\ 0.4860 & 0.5140\end{bmatrix}$	0.4860	0.5140
June	$\begin{bmatrix}0.38 & 0.62\\ 0.37 & 0.63\end{bmatrix}$	$P^5\begin{bmatrix}0.3737 & 0.6263\\ 0.3737 & 0.6263\end{bmatrix}$	0.3737	0.6263
July	$\begin{bmatrix}0.46 & 0.54\\ 0.26 & 0.74\end{bmatrix}$	$P^7\begin{bmatrix}0.3256 & 0.6750\\ 0.3256 & 0.6750\end{bmatrix}$	0.3256	0.6750
August	$\begin{bmatrix}0.43 & 0.57\\ 0.26 & 0.74\end{bmatrix}$	$P^6\begin{bmatrix}0.3133 & 0.6867\\ 0.3133 & 0.6867\end{bmatrix}$	0.3133	0.6867
September	$\begin{bmatrix}1.00 & 0.00\\ 0.30 & 0.70\end{bmatrix}$	$P^{28}\begin{bmatrix}1.000 & 0.00\\ 1.000 & 0.00\end{bmatrix}$	1.000	0.00
October	$\begin{bmatrix}0.39 & 0.61\\ 0.40 & 0.60\end{bmatrix}$	$P^5\begin{bmatrix}0.6778 & 0.3222\\ 0.6778 & 0.3222\end{bmatrix}$	0.6778	0.3222
November	$\begin{bmatrix}0.71 & 0.29\\ 0.61 & 0.39\end{bmatrix}$	$P^5\begin{bmatrix}0.6778 & 0.3222\\ 0.6778 & 0.3222\end{bmatrix}$	0.6778	0.3222
December	$\begin{bmatrix}0.90 & 0.10\\ 0.71 & 0.29\end{bmatrix}$	$P^7\begin{bmatrix}0.8765 & 0.1235\\ 0.8765 & 0.1235\end{bmatrix}$	0.8765	0.1235

Conclusion

- The Data fit perfectly well into Markov's Assumptions
- The dry spell is highest in Pre-monsoon and Post-Monsoon compared to wet and rainy spells for Uyo community
- The highest probability of rainfall occur in August, followed by July; whereas the probability of perpetual dryness is a certainty for Eket community
- Findings Reflect near real picture of the rainfall situation in the two communities

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